

COMPRESSIBILITY OF NATURAL GAS

Class 1040

Jeffrey L. Savidge, Ph.D.

“ The first requirement of gas measurement is accuracy”

D.A .Tefankjian

ISHM Lawrence Reid Award Recipient, 2000

INTRODUCTION

The accurate measurement of natural gas and natural gas related fluids is difficult. It requires care, experience, and insight to achieve consistently accurate measurements that can meet stringent fiscal requirements. It is particularly difficult to measure complex fluid mixtures that are exposed to: (1) a range of operating conditions, (2) dynamic flow and fluid property behavior, and (3) changing equipment conditions.

The compressibility factor is a ubiquitous concept in fluid measurement. It is used throughout many measurement practices and standards. At its most practical level, the compressibility factor is another fluid measurement correction factor. Unfortunately the mathematical methods, tools, descriptions and data associated with the compressibility factor obscure much of its simplicity. The purpose of this paper is to provide background on the development of the compressibility factor and related methods. It discusses their use in natural gas measurement, provides examples of the behavior of the compressibility factor, and illustrates the level of uncertainty that current compressibility factor data, methods and related property standards provide.

BACKGROUND

The theoretical foundations for flow measurement of complex fluids (e.g. gas, liquids and their mixtures) are the equations used for the conservation of mass and energy (i.e. equivalent to balancing the checkbook). The conservation equations establish theoretical flow rates. In addition to the conservation equations, mathematical correlations and correction factors are required to achieve acceptable measurement accuracy in the field.

The correlations that are used in fluid measurement may be relatively simple or complex. Correlations for the compressibility factor represent examples of the more complex ones. Acceptable measurement accuracy in the field depends on the quality of the implementation of these equations and methods relative to the operating conditions. Achieving acceptable accuracy is based on a number of considerations, e.g. (1) what can be routinely achieved in the laboratory, (2) what is practical for field practice, (3) commercial partner and customer requirements, (4) conformity with regulatory requirements, and (5) associated measurement standards prescribed by industry experience. In general, good measurement accuracy leads to independent reproducible product measurement results and provides transparency for all stakeholders, i.e. business and operations validation, and regulatory compliance.

In practice mathematical correlations are used to bind the conservation equations to laboratory measured data and related measurement equipment. Regression analysis methods are routinely used to *numerical tune both the mathematical form and the parameters* in the correlations to an underlying set of experimental data. The data must be accurate because the data have a big impact on determining the performance level of the equations. The application of such methods can be a complex process. For example, regressing or fitting an equation can severely restrict its use to a particular operating range, fluid type, or equipment characteristic. Such restrictions can lead to poor results in the field if the correlation is misapplied beyond the intended fluid, operating condition or equipment.

The types of measured data that are used to develop correlations may include: (1) equilibrium data (i.e. data obtained under static conditions, e.g. constant T, P from a laboratory), (2) relatively dynamic experimental data, such as flow rate data obtained under quasi-steady state conditions (i.e. relatively constant flow conditions with respect to time), and (3) transient flow data obtained under rapidly changing flow conditions (often associated with

field operations). The type and quality of data used to establish measurement performance depends on the application, type of measurement, and uncertainty target.

Reference quality data are an essential and important concept. They play a key part and are required to achieve high quality measurement. Reference data provide realistic flow performance validation. In general reference quality data are carefully measured data *on a realistic system* that have been obtained by multiple independent laboratories. Multiple laboratories are important because this helps to eliminate potential bias errors that may occur with a single laboratory. Highly traceable reference data provide the foundation for validating measurement results and custody transfer performance. In general, the data used to build measurement correlations, are typically measured by independent laboratories, produced by in-house laboratories, and/or measured at independent fluid flow measurement facilities. The data are often public data, i.e. non proprietary data, as they should be since the public are indirectly stakeholders in the results. High quality public data demonstrates transparency and performance. This reduces commercial and regulatory uncertainty, and hopefully, some day - will lead to reducing government regulation. Reference quality data on realistic systems raises the confidence and credibility in the use of any particular measurement method and the results it produces in the field.

Mathematical correlations are used to estimate information about the thermophysical and fluid dynamic conditions of many different types of fluids in flow lines. Any measurement application or technology that is concerned with measuring the volume, mass, or energy of a product must use some means to determine fluid density, i.e. the amount of product mass per unit volume. This may be accomplished through direct measurement of the density in the laboratory, measurement in the field and/or through the use of mathematical correlations. The compressibility factor is often defined through its relation to density. It is a particularly important quantity for natural gas and related fluid custody transfer.

EQUATIONS OF STATE

Equations of state are mathematical correlations used to relate fluid property behavior, e.g. compressibility or density, to pressure, temperature, and fluid composition. An equation of state correlation can be used in gas measurement applications to estimate the thermophysical properties and the phase condition (e.g. gas, liquid or both) of the fluid being measured.

There are many different types of equations of state. Some equations are simple mathematical functions such as the ideal gas equation, and some are quite complex functions. An example of a complex equation is the Gas Research Institute's (GRI) High Accuracy Natural Gas equation of state and the GERG2004/2008 equations. GRI's gas phase equation is distributed as A.G.A. Report No.8, A.G.A Report 10, API Chapter 14.2., It is also distributed through the International Standards Organization as ISO 12213, and ISO 20765 equations which include the GERG 2004 methods. It is expected that ISO 20765 part 2 will be extended to include the GERG 2008 method which has a broader range of application and complexity than the other methods. Like any product, each equation of state has its own utility, application and peculiarities. An equation of state used for gas measurement needs to provide values for the compressibility factor, sound speed, density and related thermophysical properties that meet or exceed the accuracy required for gas custody transfer requirements under realistic operating conditions.

The gas industry has been developing gas measurement standards for over 75 years. Technological advances and operational challenges have caused standards to continually adapt and evolve to support more efficient gas measurement technologies and new engineering / business demands. A number of measurement standards have been developed to determine the compressibility factor. Relatively recently a new A.G.A. measurement standard has been published for sound speed, A.G.A. Report No.10 and the GERG 2004 work in ISO 20765. Both of these methods use key parts GRI's equation of state related work. Speed of sound methods are based on the equation of state used to predict compressibility factor equations. Some current as well as historical methods and standards for compressibility and sound speed are listed below.

1. AGA Report No.8, "Compressibility Factors of Natural Gas and Other Related Hydrocarbon Gases"¹
2. International Standard, 12213 Natural Gas – Calculation of Compression Factor²

3. Manual for the Determination of Supercompressibility Factors for Natural Gas – NX19³
4. API MPMS Chapter 14.2⁴
5. GPA 2172-96 Calculation of Gross Heating Value, Relative Density, and Compressibility Factor for Natural Gas Mixtures from Compositional Analysis⁵
6. A.G.A. Report No.10, Speed of Sound in Natural Gas and Other Related Hydrocarbon Gases⁶
7. International Standard, ISO - 20765, Natural Gas – Part 1 and Part 2. Calculation of Thermodynamic Properties⁷

HISTORICAL PERSPECTIVE ON THE COMPRESSIBILITY FACTOR

Modern work on the pressure-volume-temperature (PVT) behavior of gases can be traced to pre-industrialized Europe. In the 17th century (1660) Robert Boyle and Edme Mariotte measured the PVT behavior of air at low pressures and established the inverse relationship between pressure and volume at constant temperature, $PV = k_T$ (constant). This is known as the Boyle-Mariotte Law.

Over a century passed before more experimental measurements on the PVT behavior of air were made. Jacques-Alexander Cesar Charles and Joseph Louis Gay-Lussac in the period from 1787 to 1803 helped pioneer hot-air ballooning in France. They worked independently and established the law of equal thermal expansions of gases at constant pressures, $V/T = k_p$ (constant).

In 1834 Emile Clapeyron recognized that the equations of Boyle-Mariotte and Charles-Gay-Lussac could be combined to create an equation (equation of state) for a perfect gas $PV = R_o T$ where R_o was a gas dependent constant. This equation linked pressure, volume and temperature. A milestone was achieved in 1845 when Victor Regnault cast Clapeyron's perfect gas equation into ideal or "perfect" gas equation form. He did this by applying Avagadro's hypothesis on the volume occupied by one mole of an ideal gas, i.e. $PV = nRT$. In the ideal gas equation n is the number of moles of gas and R is a constant called the universal gas constant which is independent of gas type. This was the final relationship required to permit generalizing (predicting) the PVT behavior of all substances that behave as ideal gases, i.e. gases at very low pressures and high temperatures.

In addition to the work on the PVT behavior of air, modern work on the sound speed in air can be traced to 1738. Sir Issac Newton indirectly surmised the relation between the sound speed to the square root of the ratio of the pressure and density. At that time measurements on the sound speed in air were within 1% of present day measurements. Following the work of Charles and Gay-Lussac, more interest was expressed in sound speed. Jean-Baptiste Biot in 1802 expressed the sound speed as $v = (P_o (1+k)/\rho_o)^{1/2}$, where v is the sound speed, P the pressure, ρ the density, and k a constant. This was followed in 1845 by Sir George Gabriel Stokes who analyzed the fluid dynamic propagation of sound waves.

Shortly after Regnault formulated the ideal gas equation it was realized that the ideal gas equation did not predict measured results for high pressure conditions. Consequently research focused on ways to improve the ideal gas equation. Pioneering measurements by experimentalists such as Andrews, Amagat and others established that the ideal gas equation had errors that ranged from a few percent at low pressures to over 70 percent at high pressures. Experimental data from Andrews and others provided the experimental data foundation necessary to advance beyond the simple ideal gas equation.

Early experimentalists recognized that a new gas law equation was needed to account for the large discrepancies between values obtained from the ideal gas equation and values measured in their laboratories. In other words, a new, more accurate gas law equation was needed to describe the behavior of real gas molecules such as those found in natural gases and related petroleum fluids. The direction and magnitude of the deviation from ideal gas behavior for methane and hydrogen are illustrated in Figure 1 and Figure 2. The figures show the compressibility factor vs. pressure at various temperature and pressures. The ideal gas behavior occurs at a compressibility factor value equal to one for all conditions. Real gas behavior is illustrated by the curves on each chart.

Researchers later established that the ideal gas equation had a number of underlying assumptions and flaws that were only exposed when higher density conditions (e.g. high pressure) were encountered. The flaws in the ideal gas equation led to erroneous calculation results at high pressure conditions. The basic assumptions in the ideal gas equation are: (1) gas molecules are infinitesimal spheres that do not occupy any volume, (2) collisions between the molecules and wall are completely elastic, and (3) that there are no attraction or repulsion forces exerted between gas molecules. The assumptions are not valid for real molecules at real operating conditions; nevertheless, the ideal gas equation provided the basis for the development of important ideas. The ideal gas equation ideas were needed to construct proper mathematical formulations of the behavior of real fluids. In order to improve theoretical calculations it was necessary to correct the assumptions that underlay the ideal gas equation of state.

J.D. VAN DER WAALS

The goal in early equation of state development was for theoretical calculations to agree with the measured data under the pressure conditions of the real-world. The gas compressibility factor was conceived by van der Waals as the means to account for the deviation of real gas behavior from the behavior of an ideal gas. It was used to provide a measure of the difference between the real volume that is occupied by a gas at a given temperature and pressure, and the volume obtained under the same conditions from the ideal gas equation - in other words, it is a simple correction factor.

In 1873 J.D. van der Waals proposed his idea on the continuity of the vapor and liquid states. This was followed with his work on a corresponding states theorem on the behavior of real gases. Van der Waals proposed terms to Regnault's ideal gas equation ($PV=nRT$) to correct the generalized ideal gas equation. Van der Waals added additional terms to both the P (pressure) and V (volume) variables in the ideal gas equation. His new terms were correction terms that accounted for the effect of molecular attraction forces on the pressure term, P , and accounted for the size of real molecules on the volume term, V . Van der Waals idea was to simply replace P in the ideal gas equation with $(P + a/v^2)$ and V in the ideal gas equation with $(V - b)$, where his new " a/v^2 " term increased the pressure so it was closer to the observed (measured) pressure, and his " b " term corrected the ideal gas volume term for the actual volume occupied by the molecules. Van der Waals new equation was a great scientific success! It qualitatively described the PVT behavior of real fluids and improved the accuracy of calculated volumes to measured volumes. A key feature that emerged from his new equation was its mathematical functional form, i.e. a cubic form (three term polynomial in density). This functional form provided the necessary mathematical features required to qualitatively capture the gas-liquid phase boundary.

Van der Waals' work was the first qualitatively accurate description of real fluid behavior beyond the ideal gas equation. After the new equation of state was developed, van der Waals proposed his corresponding states concept. This concept states that all molecules, e.g. methane, ethane, propane, etc., exhibit approximately the same thermophysical property, e.g. compressibility (Z or F_{pv}), behavior at similar reduced conditions. Reduced conditions include the reduced temperature, T_r , the volume (or density), V_r , and the reduced pressure, P_r . The reduced temperature is defined by $T_r = T/T_{critical}$, the reduced pressure is defined by $P_r = P/P_{critical}$, and the reduced $V_r = V/V_{critical}$. Van der Waals' insight and corresponding state concept generalized the behavior of many fluids by using the reduced temperature, reduced pressure, and reduced volume. He provided the keystone required to begin the process of generalizing fluid property behavior for many of the chemical substances found in natural gases and related fluid mixtures, as well as most industrial fluids. Van der Waals received the Nobel Prize in 1910 in physics for his significant insight and contributions to our understanding of how real fluids behave.

VAN DER WAALS' DESCENDANTS

Van der Waals ideas can be found in virtually every engineering equation of state in use today. They are particularly prevalent in equations of state in the energy and chemical process industries. Many modern equations of state, including GRI's Detail Method in A.G.A. Report No.8, and more recent variations continue to retain van der Waals basic ideas through the application of component and mixture characteristic parameters.

A large number of equations of state have followed van der Waals pioneering efforts. In general, subsequent equations of state have been constructed or modified by adding different equation terms to permit modeling

increasingly complex molecules and to improve the agreement with measured data. Some equation of state modifications were empirical, i.e. not based on physical theory, and some utilized a theoretical foundation, i.e. modifications based on the physics and chemistry of real molecules.

Many modifications have been made to the original van der Waals equation for fluid behavior. A more extensive theoretical foundation has been established that extends van der Waals ideas to include the use of molecular based parameters. Modern embellishments to his ideas revise the original equation terms and incorporate more detail on the molecular structure of fluids. This has helped to improve the accuracy and broaden the application range of van der Waals type equations of state. Modern computer based non-linear data regression analysis permits the rapid evaluation of virtually any functional form beyond the van der Waals functions to adequately fit experimental data over the entire PVT surface. This approach is reflected in many modern equations of state.

In 1955 Pitzer proposed one of the most successful modifications of van der Waals corresponding states theorem¹⁶. Pitzer proposed a functional form that could account, to some extent, for molecular shape differences (i.e. accentricity) on the physical properties. The quantity is called the Pitzer accentric (non-spherical molecule shape) factor. It has a strong theoretical basis and is widely used in many equation of state modeling efforts. His ideas are captured in many recent equations as a factor called the "shape factor".

Historically important equations of state for the gas industry include the Onnes virial equation (1885, 1901, 1927)⁸, Beattie-Bridgeman equation (1927)⁹, Benedict-Webb-Rubin (BWR) equation in 1940¹⁰, Redlich-Kwong (1949)¹¹, Lee and Kesler (1975)¹¹, and the Peng-Robinson (1976)¹². The Redlich-Kwong as modified by Soave (SRK) in 1972¹² and the Peng-Robinson (PR)¹³ equations are highly successful variations of the original van der Waals formulation. The BWR equation was the foundation for the NX-19 method³. In addition to the above equations a modification of the original BWR equation (MBWR) by Han and Starling (1972)¹⁵, has been successfully used for a wide range of hydrocarbons in production and process engineering.

The generalized cubic equations, i.e. SRK and PR, are attractive because of their compact functional forms (like van der Waals original equation) and generality. This permits easy understanding, efficient computer calculations for many applications, including oil reservoir and chemical processing applications where many computational intensive phase calculations are required. Unfortunately generalized cubic equations are mathematically constrained -i.e. choked. They do not appear to be able to deliver high accuracy for compressibility values given high quality input information. They often produce inaccurate results for many gas measurement applications. The LK, PR, SRK and MBWR have been used extensively as engineering design equations of state in gas production, and in petroleum and chemical processing applications. In general their accuracy is more than sufficient for such engineering applications. The virial equation, though limited in its density range, is also highly successful. It is used for low to moderate pressures in GPA 2172 and in A.G.A. Report No.8 / API MPMS 14.2.

The GRI / AGA 8 / API 14.2 equation of state is a semi-empirical modeling approach for the thermophysical properties. It incorporates both theoretical and empirical elements. This has provided the correlation with the degree of numerical flexibility needed to achieve high accuracy for compressibility, sound speed and other thermophysical properties over a broader range of conditions than had been previously possible.

GAS INDUSTRY MEASUREMENTS, SUPERCOMPRESSIBILITY AND COMPRESSIBILITY FACTORS

For many years the most accurate way to determine the compressibility factor of natural gas was by direct measurement in the laboratory. In 1916 and 1917 publications by G.A. Burrell and I.W. Robertson of the Bureau of Mines, and by R.F. Earhart and S.S. Wyer of Ohio State University described an apparatus for determining the compressibility of natural gas¹⁶. This was followed by the work between 1926 to 1930 by Howard S. Bean of the National Bureau of Standards (NBS) who supervised the development of a PVT apparatus and the first gas industry efforts to evaluate the effect of the compressibility factor in orifice meter measurement. The original NBS apparatus became known as the Bean apparatus. There was also a modified version of the Bureau of Standards apparatus which was called the Beckman apparatus. Both of the devices determined the gas law deviation factor, i.e. compressibility factor, through successive volume measurements.

The early laboratory devices provided a direct measurement of the deviation factor from ideal gas behavior and were based on the work of van der Waals. The early compressibility factor devices required an accurate determination of the sample volume. Determining the sample volume tended to be tedious and time consuming

for the operators in the laboratory. A more convenient compressibility factor measurement technology was sought.

In 1936 E.S. Burnett developed a new device to measure compressibility factor for the Bureau of Mines¹⁷. The method relied on pressure measurements. It had the advantage of not requiring accurate measurement of volumes and was more convenient to use in the laboratory than earlier equipment. This device was manufactured and marketed to the gas industry by the Refinery Supply Co. and then Chandler Engineering. It found widespread use in gas measurement laboratories and industry research laboratories.

AGA Report No.2 *required the determination of supercompressibility factor by measurement*. Early experimental compressibility studies by the gas industry obtained PVT data up to 500 pounds pressure per inch. The deviation from the Boyle-Mariotte Law at the time was referred to as supercompressibility. Supercompressibility was mathematically defined as approximately equal to inverse of the square root of Z, i.e. $1/\sqrt{Z}$. The supercompressibility factor corrected the final volume at typical operating conditions between three to six percent. At extreme conditions the correction to the volume was as much as twenty percent.

In the time period of the 1930's and 1940's, the California Natural Gasoline Association, using data from Standing, Katz, Brown and Holcomb from the Natural Gasoline Association of America(now the Gas Processors Association), found the supercompressibility factor could be correlated to the temperature, pressure and specific gravity. They later obtained supercompressibility data above 500 psig. This became the basis for making the first calculations of supercompressibility factors in place of actual laboratory measurements. Experimental tests in gas company laboratories later showed that the high pressure PVT data did not agree with gas company laboratory data. The need for high quality realistic reference data was apparent.

Laboratory measurements of deviation data (supercompressibility determination) on pipeline company gas mixtures were common from the 1930's into the 1970's. Natural gas company customers required measurements of the deviation factor in their contracts for their specific gas mixtures. These tests were generally performed in the gas company laboratory. The Burnett apparatus was the experimental method of choice for gas companies to determine the deviation factor or supercompressibility factor. Much of the old laboratory data has been lost as the industry has evolved away from laboratory measurements.

Additional laboratory tests on natural gas supercompressibility were made by Samuel R. Beitler of Ohio State University in 1951-1953 and published as Basic Supercompressibility Tables in 1954. PAR Research Project NX-19 was initiated in 1956 to extend the original Supercompressibility Tables. This work was completed in 1960 by Richard Zimmerman and John Boyd of Ohio State University. One of the significant outcomes from this work was the creation of an equation for the calculation of the supercompressibility tables. This became the first widely accepted method for computer calculation for gas measurement. It provided the foundation to transition from gas company laboratory measurement of supercompressibility using devices, such as the Burnett apparatus, to an analytical determination of supercompressibility using an equation of state. The equation of state approach permitted the direct use of field measurements of pressure, temperature and composition.

The NX-19 method developed by Zimmerman from Ohio State applied a truncated version of the original BWR. The BWR equation was originally developed at Massachusetts Institute of Technology. This approach extended the ideas of J.D. van der Waals and Kammerlin Onnes by adding higher order density and exponential terms to the virial equation. The functional form of Zimmerman's equation was a mathematical equation truncation of the BWR equation. It used auxiliary fitting functions for different regions of the pressure-temperature surface, i.e. different equations for different operating conditions. The auxiliary fitting functions improved the accuracy of the BWR compressibility factor calculations over specific P-T regions.

The NX-19 method, i.e. truncated BWR method, produced reliable supercompressibility factors, F_{pv} , for high methane natural gas mixtures at typical pipeline conditions. Because of the application at the time and NX-19's improved accuracy, it became an essential part of the natural gas industry's orifice measurement calculation procedure beginning from about 1960 to the mid 1990's. The NX-19 method found widespread use by gas companies throughout the world.

The NX-19 method, although limited, has an important technological legacy in modern measurement practice. One of the significant problems with the NX-19 equation was that the formulation contained discontinuities in the

derivatives of the compressibility factor at the boundaries of the auxiliary fitting functions. This limited its engineering utility to supercompressibility factor calculations. The equation could not be used to meet other gas engineering needs including critical flow nozzles calculations, sound speed calculations, or compressor calculations. In addition, the NX-19 method was biased for rich gases and high dilutes content gases. The concern over bias errors caused the industry to initiate the development of a new equation of state method to replace the NX-19 work. The new work was done by GRI and later became A.G.A. Report No.8, API 14.2, ISO 12213 and portions of ISO 20765.

GAS RESEARCH INSTITUTE'S HIGH ACCURACY EQUATION OF STATE FOR NATURAL GASES

In 1980 the Gas Research Institute (GRI) undertook an effort to improve upon the NX-19 method for custody transfer. The scope of the work was to improve the capabilities for calculating the compressibility factors significantly beyond the temperature, pressure, and composition ranges of PAR Project NX-19. This work was under the direction of Prof. K.E. Starling at the University of Oklahoma, Gas Research Institute, and the AGA Transmission Measurement Committee along with a number of independent advisors.

The new equation of state modeling effort required high accuracy data. Over 34,000 PVT data points were collected from the published literature and industry sources. The data included pure, binary, multicomponent, and natural gas mixtures. The data were rigorously evaluated using a variety of methods. Only a fraction of the data met the modeling uncertainty criteria required for model development and equation testing purposes. In 1985 the first generation of GRI'S high accuracy equation of state for natural gases was implemented. A.G.A. published it at that time to replace the NX-19 method. This was the first edition of A.G.A Report No.8.

Concurrent with the publication of the first edition of A.G.A. Report No. 8, the equation of state method was expanded to compute other important physical properties such as the sound speed and related thermophysical quantities. In 1985 GRI's first comprehensive high accuracy thermodynamic model of the physical properties of natural gas and related fluids computer program was completed. It was entitled SuperZ and provided accurate sound speed, critical flow factors, and other related thermophysical property calculations^{16,17,18}. It significantly extended the 1964 and 1972 work of Johnson on critical flow factors. Johnson's original work utilized the BWR equation of state method. SuperZ provided a set of equations, algorithms, and FORTRAN code for computing all equation of state quantities for gas metering. SuperZ achieved in 1985 what was later to be identified in the late 1990's as the objectives for A.G.A. Report No.10^{19,20,21} and is imbedded in ISO 20765 parts 1 and 2. The original 1985 work was based on a very limited set of experimental data that was available at the time.

Subsequent revisions of the equations that were developed by GRI at the U. of Oklahoma for A.G.A. Report No.8 have occurred. These versions provide incremental improvements on the approach used to model natural gases with respect to compressibility factors. The data show that it is applicable to virtually all natural gases found in gas pipelines at typical operating conditions. The revisions have incorporated a broader range of natural gas data and have been adopted by A.G.A., A.P.I., G.P.A. and ISO. In 2002 A.G.A. completed the preparation of A.G.A. Report No.10 to compute sound speed. A.G.A. Report No.10 is based on the compressibility equations from A.G.A. Report No.8. The GERG, a European research consortium of gas companies, undertook an extensive effort in the mid to late 1990's, to extend the application of high accuracy equations beyond the gas phase properties to include the critical region and phase boundary predictions for dew points in addition to the gas phase compressibility factor and other properties prediction capabilities. This effort was beyond the original scope identified by the AGA Transmission Measurement Committee for the development of AGA Report No.8 and API 14.2 which concentrated solely on the single gas phase.

The missing element in all of this work remains the need for high quality mixture reference data for real gas mixture phase behavior. GRI initiated an effort to fill this data gap in the late 1995. Regulatory and industry changes resulted in a change to GRI's research direction. In the meantime, the GERG modeling work continued and has produced results that look promising for providing a tool that can be used to provide high quality compressibility results that are comparable in most cases, and better in some uncommon cases, than the current AGA8 and API 14.2. In addition the results show the capabilities to model phase behavior, although the uncertainty on real multicomponent fluid mixtures has not been provided or known. Again, realistic reference data

are needed. It is expected that the results will be implemented through the International Standards Organization with possible adoption of the method in the U.S by AGA and API at some point in the future.

COMPRESSIBILITY FACTOR

The compressibility factor, Z , is defined as the ratio of the real gas volume to the ideal gas volume. The real gas volume and ideal gas volume are determined at the same temperature, pressure and gas composition. The real gas volume is the measured volume of a given mass of gas at an equilibrium temperature and pressure. The ideal gas volume is a calculated volume at the same temperature, pressure, and gas composition as the measured volume.

$$(0.1) \quad Z = \frac{V^{real}}{V^{ideal}}$$

$$(0.2) \quad V^{real} = V_{T,P,n}^{measured}$$

$$(0.3) \quad V^{ideal} = \frac{nRT}{P}$$

where

n = number of moles (mass)
 R = gas constant
 T = temperature, absolute
 P = pressure, absolute
 V^{real} = real gas volume
 V^{ideal} = ideal gas volume
 Z = compressibility factor

A comparable way to define the compressibility factor is

$$(0.4) \quad Z = \frac{P\bar{V}}{RT}$$

where \bar{V} = is the real molar volume (inverse of the molar volume is the molar density).

The real gas compressibility factor corrects the volume relative to the assumptions in the ideal gas equation. A real gas that behaves as an ideal gas has a compressibility factor value of 1, this generally occurs at low pressures and high temperatures. Figure 1 illustrates the variation in values for methane's compressibility factor. The compressibility factor chart shows a wide range of temperature and pressure conditions. Each curve is at constant temperature. The temperature range illustrates the behavior of methane's compressibility factor at conditions for liquefied natural gas, gas processing, surface exposed pipe conditions, transmission lines, and well conditions.

Figure 1 shows that as methane is compressed, molecular effects reduce the volume occupied by the real gas relative to the ideal gas volume, i.e. the real to ideal volume ratio is less than 1 for eqn. 1. This results in compressibility factors that are less than 1. The amount of compression in the volume depends on the chemical component, temperature and pressure. As pressure is further increased, repulsive forces between molecules begin to dominate. This leads to a reversal in the compressibility factor value and results in an expansion in the real to ideal volume ratio as pressure is further increased.

Methane Compressibility Factor Behavior Example

Low to high pressure range

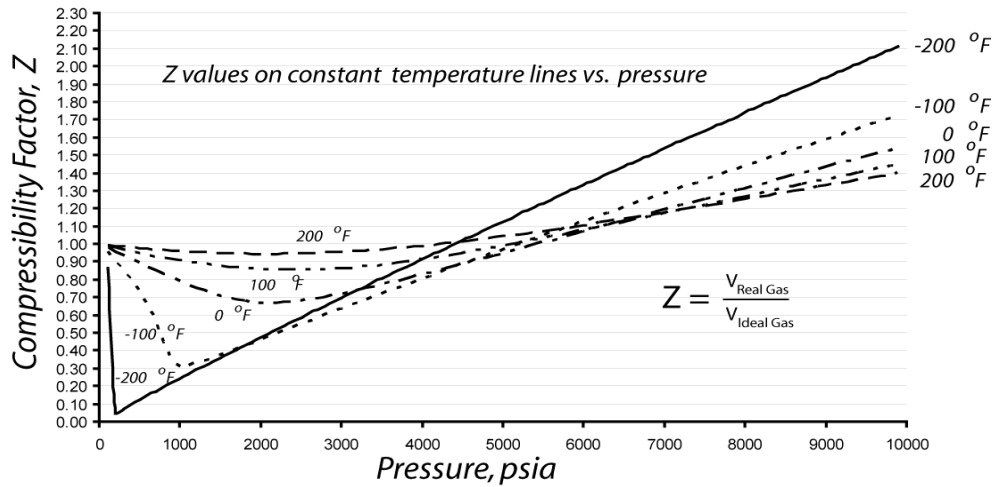


Figure 1. Methane compressibility factor example, low to high pressure range.

Most of the chemical components that make up natural gas mixtures follow the general type of behavior as shown in figure 1. An example of an uncommon natural gas component that does not follow the behavior of methane and other hydrocarbons is illustrated in figure 2. Figure 2 shows the behavior of the compressibility factor for pure hydrogen. The compressibility factor is plotted over the same temperature and pressure range as in figure 1. In this case the compressibility factor is greater than 1 regardless of the T,P conditions.

Hydrogen Compressibility Factor Behavior Example

Low to high pressure range

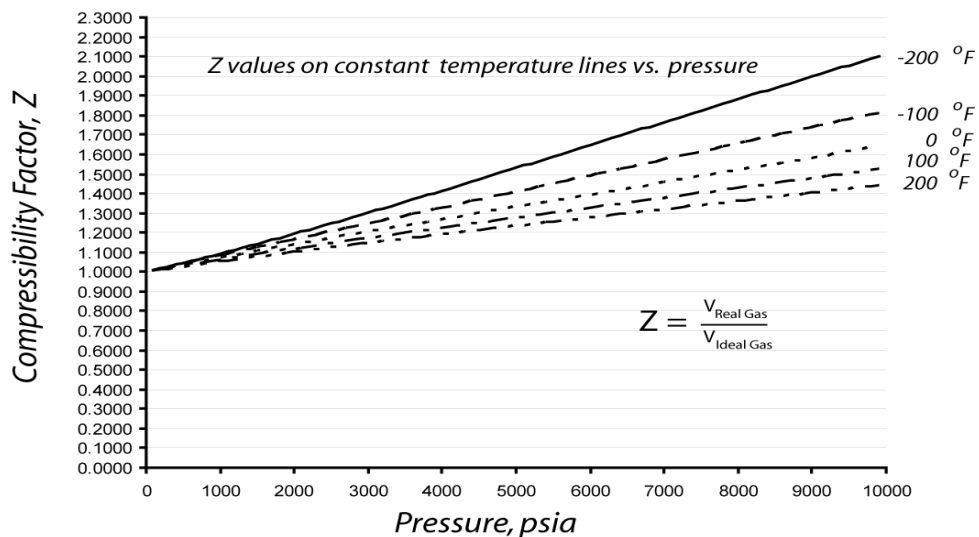


Figure 2. Hydrogen compressibility factor example, low to high pressure range.

For typical natural gases at natural gas measurement conditions the compressibility factor corrects the ideal gas volume from less than one percent at near atmospheric conditions to about twenty percent at high pressure conditions. Consequently it is an important quantity that must be considered in gas measurement. It is required to account for changes in gas volume or density caused by changes in the temperature, pressure and composition of the gas. The magnitude of the contribution of the compressibility factor to the overall error in measurement depends on the method that is used to make the measurement, e.g. orifice meters, turbine meters, or ultrasonic meters.

Figure 3 focuses on a small portion of figure 1 to show in more detail the range of compressibility factor values for pure methane on constant temperature curves for pressures up to 2000 psia. The constant temperature curves in figure 3 between 0 °F and 100 °F cover most gas phase measurement conditions.

Natural gas mixtures are mostly methane. It is the most important component for calculating the compressibility factor of a natural gas mixture. Other hydrocarbon components, e.g. ethane, propane, etc. and diluents found in natural gases also contribute to the overall compressibility factor value for real mixtures. As the components in a mixture change, there is a corresponding change in their contribution to the compressibility factor. For example, for a given mixture consisting of methane and hydrogen at constant temperature and pressure, the compressibility factor value of the mixture will be a composition weighted value that falls between the values given in figures 1 and 2. In general it is not a simple average of the concentrations of each component in the mixture.

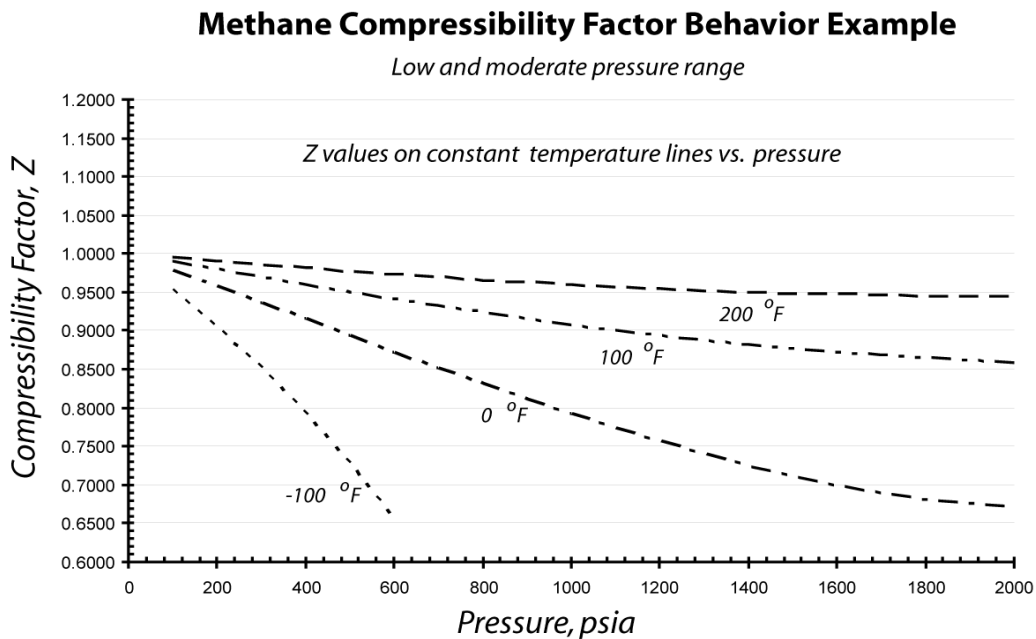


Figure 3. Methane compressibility factor example, low and moderate pressure range.

Mixtures of hydrocarbons follow the same basic trends, to a greater or lesser degree as shown in figures 1 and 3. The variation depends on the composition. In order to accurately model the variation in the compressibility factor for gas mixtures, the Gas Research Institute (GRI) developed the high accuracy equation of state for natural gas compressibility factor with all the supporting equation(s). GRI's results are distributed by A.G.A., A.P.I., and I.S.O. They go under different equation names based on who distributes them, e.g. A.G.A. Report No.8 and Report No.10. A.P.I. Chapter 14.2, ISO 12213 Part II and ISO 20765 Part I.

Equation 5 presents GRI's high accuracy equation of state for natural gas mixtures which is present in A.G.A. Report No. 8 Detail Method to compute the compressibility factor. The A.G.A. 8 equation is also the same

equation of state used in A.G.A. Report No.10 to compute speed of sound and related thermophysical property calculations for natural gases and related fluids. The equation is provided here only for illustrative purposes. The terms are left undefined. The interested reader is referred to A.G.A. Report No.8 and API MPMS Chapter 14.2 for details regarding the specific terms and variables that are used in the equation.

The compressibility factor equation appears complex. Technically it is just a mathematical correlation that models a large collection of natural gas related PVT data. In general the equation is used as representation of the best experimental PVT data to the measurement uncertainty level of the PVT data.

The number of terms and parameters in the equation were determined from a regression analysis of the experimental compressibility factor data. The data are the foundation of the correlation. The equation is an application and extension of the historical concepts discussed in the early part of this paper. The historical ideas have been extended and applied to high quality data on natural gas components and their mixtures.

A significant portion of the complexity in the Detail Method in A.G.A. Report No.8 or any high accuracy equation of state method arises out of the requirement to achieve high accuracy for mixtures. Non-linear regression algorithms used to develop the equation of state introduce equation terms to support achieving the overall target uncertainty levels (see A.G.A. Report No.8 for the details). It assumes that the regression data is of sufficient quality for the terms in the equation to be statistically significant. In addition, the use of many model parameters permits the model to be applied over a broad range of fluid measurement conditions and properties of interest.

The functional form of any correlation is determined, in part, by the underlying experimental data that are regressed to build the model, in this case the regression data was pure, binary and multicomponent hydrocarbon PVT data.

$$(0.5) \quad Z = 1 + dB - dK^3 \sum_{n=13}^{18} \frac{C_n^*}{T^{u_n}} + \sum_{n=13}^{58} \frac{C_n^*}{T^{u_n}} (b_n - c_n k_n (dK^3)^{k_n}) (dK^3)^{b_n} \exp(-c_n (dK^3)^{k_n})$$

The A.G.A. Report No. 8 expression has embedded in it calculations for mixtures. The details of the mixture calculations are not shown here. The mixture functions are mathematical relationships that permit predicting mixture fluid properties by numerically nudging pure fluid correlation parameters. They are further refined through the use of fluid specific regression interaction parameters. These quantities are known as binary interaction parameters. Binary interaction parameters are common in virtually all equations of state. They often require the regression of experimental binary mixture data to improve the description of fluids containing the binary components. A key element in the A.G.A. Report No. 8 equation of state is its ability to accurately compute the properties for real natural gas mixtures. It accomplishes this through the use of hybrid mixing rules. The mixing rules provide a practical means to describe fluid mixture behavior. It should be noted that the equation was not developed for the high density fluid critical region. In most gas measurement applications the operating conditions are far removed from the critical region.

A major difference in the AGA 8 equation of state versus the cubic type equations of state of van der Waals and others is in the development of the equations. AGA8 made extensive use of modern regression analysis tools to develop the equation of state. Advanced regression analysis tools coupled with modern computational capabilities provide the mathematical tool-bench needed for the development of very high accuracy equations of state. A drawback of improved regression methods is that additional equation parameters become necessary as more complex fluids are regressed over increasingly large P-T regions, e.g. into the liquid region and critical region. The physical meaning of the equation and its terms become obscured in the regression process. Table 1 provides gas mixture fluid characteristics for compressibility factor calculations using the A.G.A. Report No.8.

Quantity	Normal Range	Expanded Range
Relative Density	0.554 to 0.87	0.07 to 1.52
Gross Heating Value	477 to 1150 Btu/scf	0.0 to 1800 Btu/scf
Gross Heating Value	18.7 to 45.1 MJ/m ³	0.0 to 66 MJ/m ³
Methane (mole %)	45.0 to 100.0	0.0 to 100.0
Nitrogen	0.0 to 50.0	0.0 to 100.0
Carbon Dioxide	0.0 to 30.0	0.0 to 100.0
Ethane	0.0 to 10.0	0.0 to 100.0
Propane	0.0 to 4.0	0.0 to 12.0
Butanes	0.0 to 1.0	0.0 to 6.0
Pentanes	0.0 to 0.3	0.0 to 4.0
Hexanes	0.0 to 0.2	0.0 to Dew Point
Helium	0.0 to 0.2	0.0 to 3.0
Hydrogen	0.0 to 10.0	0.0 to 100.0
Carbon Monoxide	0.0 to 3.0	0.0 to 3.0
Argon	0.0	0.0 to 1.0
Oxygen	0.0	0.0 to 21.0
Water	0.0 to 0.05	0.0 to Dew Point
Hydrogen Sulfide	0.0 to 0.02	0.0 to 100.0

Table 1: Gas Mixture Characteristics for A.G.A. Report No.8 Method

The characteristic ranges in Table 1 were established by key industry representatives of the AGA Transmission Measurement Committee. It also included the American Petroleum Institute's measurement committee representative Mr. R. Beatty among many others. For some of the fluids in Table 1 the ranges provide approximate estimates of when certain hydrocarbon liquids will appear at a particular operating condition. For example, the 0.2 mole percent limit on hexane is roughly the maximum concentration of hexane at 60°F. The amount of hexane in the gas phase is a function of operating conditions. Liquid formation will disrupt the quality of any gas measurement, both in the laboratory and in the field.

Extensive reference data for mixtures that cover all fluid ranges was not practical at the time AGA8 was developed. In fact, it was not considered necessary by the AGA TMC and API at that time. AGA Report No.8 was not designed to apply to liquids, it was not designed to apply at hydrocarbon dew points, nor was it designed to apply in the critical region.

To obtain a compressibility factor from A.G.A. Report No.8 the equations must be solved on a computer by applying well-established iterative techniques to converge to the correct answer. Some of the iterative techniques can be very simple and some relatively complex. The complexity is driven by the need to avoid numerical solution problems that can arise in iterating to the solution. Computer programs used to compute the compressibility factor and sound speed are available in the standards as well as validation examples - AGA Report No.10 provides a reasonable example for how validation be done. GRI's Technical Reference document provides another good example (see reference list).

COMPRESSIBILITY FACTOR UNCERTAINTY

Over 34,000 experimental data points were originally analyzed²³ in order to establish the uncertainty of the A.G.A. Report No.8 equations and to compare its performance to other methods such as the NX-19 method. The analysis is being extended to the GERG04 and GERG08 methods. In the analysis, compressibility data for natural

gas mixtures were obtained from many different high quality experimental sources and organized into different categories. The data include:

- (1) reference data that were obtained from an extensive international round-robin test among four laboratories,
- (2) natural gas mixture data from Europe, i.e GERG data bank,
- (3) independent data measured at Ruhrgas,
- (4) data from the open literature.

It can be very difficult to obtain low uncertainty data in the laboratory. Problems can occur. Such problems increase the uncertainty in the experimental data. Despite, such problems, the best data to date that has been obtained for mixtures are listed reference in item above (1), reference 25 at the back of this paper. That reference provides a comprehensive list of nearly 300 realistic natural gas mixtures and associated data. The work was accomplished in 1995 by the Gas Research Institute's Thermophysical Properties Research Program under the author.

Data analysis was conducted for many of the pure and binary fluid components. These are the molecular level building block components that are used to engineer mathematical correlations for natural gases or any type of fluid mixture.

Table 2 summarizes the key mixture uncertainty comparisons for key natural gas mixtures. The uncertainties apply to Region I of A.G.A. Report No.8 and API 14.2. It is the custody transfer region. In AGA8/API 14.2 a Lean Gas is a high methane content natural gas such as a Gulf Coast Type gas mixture, a moderate gas is a Amarillo Type natural gas mixture, a rich gas is a Ekofisk Type natural gas mixture. The compositions follow those listed in AGA8/API 14.2 and its Technical Reference Document for AGA Report No.8 (GRI -93/0181, March 1995).

Table 2. Comparison of Compressibility Factor Methods for Different Gases

Gas Group / Method	Method	Average Error %	Max Error %
Lean Gas (\approx 95% methane gas)	A.G.A. 8		
	-Detail Method	0.02%	-0.14%
	-Gross Method	0.02%	-0.14%
	NX-19	0.10%	-0.62%
Moderate Gas (\approx 90% methane gas)	A.G.A. 8		
	-Detail Method	0.03%	0.06%
	-Gross Method	0.04%	0.05%
	NX-19	0.14%	0.24%
Rich Gas (\approx 85% methane gas)	A.G.A. 8		
	-Detail Method	0.03%	0.25%
	-Gross Method	0.04%	-0.32%
	NX-19	0.59%	-2.06%
High Nitrogen	A.G.A. 8		
	-Detail Method	0.06%	-0.20%
	-Gross Method	0.04%	-0.15%
	NX-19	0.13%	-0.45%
High Carbon Dioxide	A.G.A. 8		
	-Detail Method	0.03%	0.16%
	-Gross Method	0.04%	0.15%
	NX-19	0.28%	-1.04%

Comparisons in Table 2 were made by grouping the gases into six groups based on the concentration range of some components in the gas. The groupings help to illustrate the potential impact of various components on the modeling. The uncertainty of compressibility factors calculated using either of the methods in A.G.A Report No.8

methods, the Detail Characterization Method or the Gross Characterization Method, depend upon the natural gas composition and the temperature-pressure condition of the calculation.

Figures 4 through 9 provide calculation comparisons between AGA Report No.8 and the GERG 2004/2008 method(s). The comparisons on these reference natural gas mixtures show the two methods agree within the statistical uncertainty of the experimental data (in general less than 0.05%) at the most significant natural gas custody transfer conditions. The high quality data were acquired in the 1990's. Subsequent data has been obtained on very rich gases which may affect the degree of agreement between the methods and the data.

In general, the figures clearly show there is a high degree of agreement between AGA Report No.8 (1992) and the GERG (2004/2008) methods for common natural gas mixtures that were considered by AGA Transmission Measurement Committee (TMC) and API Fluid Measurement Committees back in 1992 when AGA Report No.8 was last revised. It should be noted that the 1992 revision to AGA Report No.8 was completed in less than 18 months per the AGA TMC's and API's objectives. Those objectives were strictly focused on the gas phase !

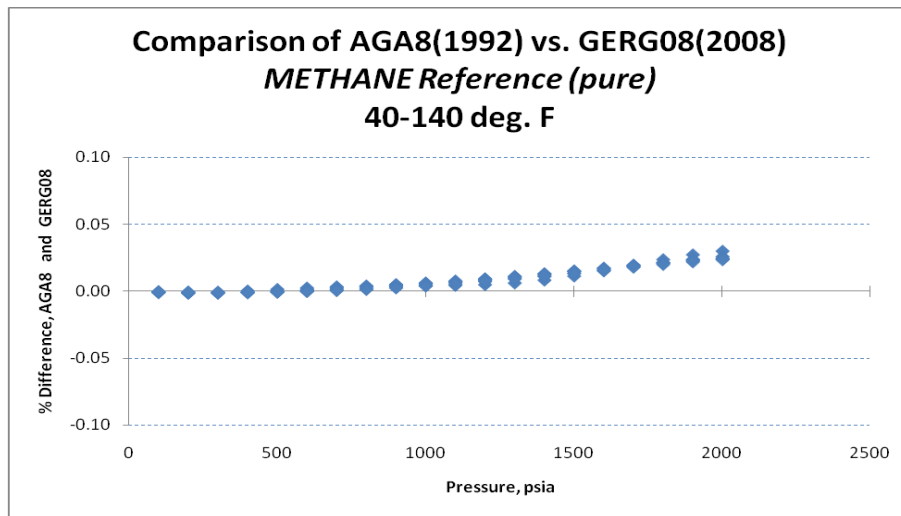


Figure 4. Methane Uncertainty - Difference between AGA8 (1992) and GERG(2004/2008)

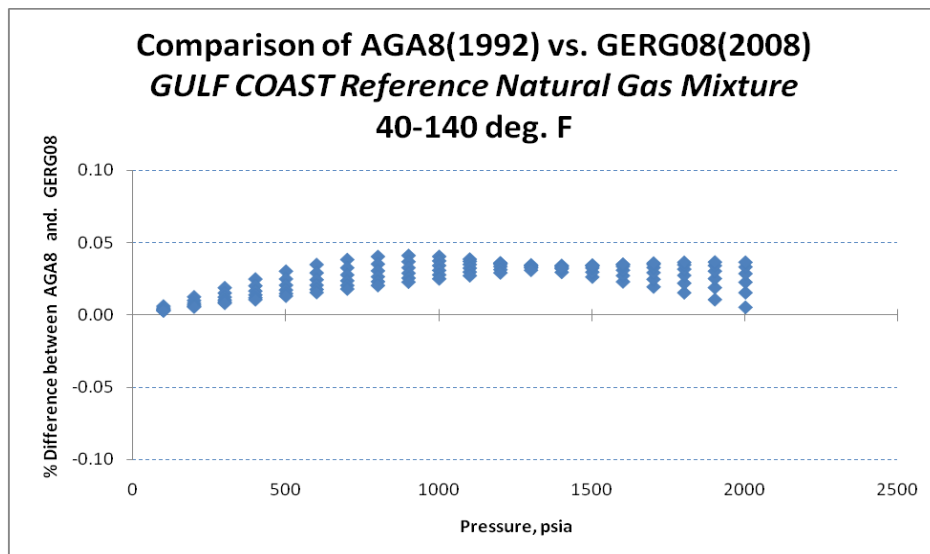


Figure 5. Gulf Coast Gas Uncertainty - Difference between AGA8 (1992) and GERG(2004/2008)

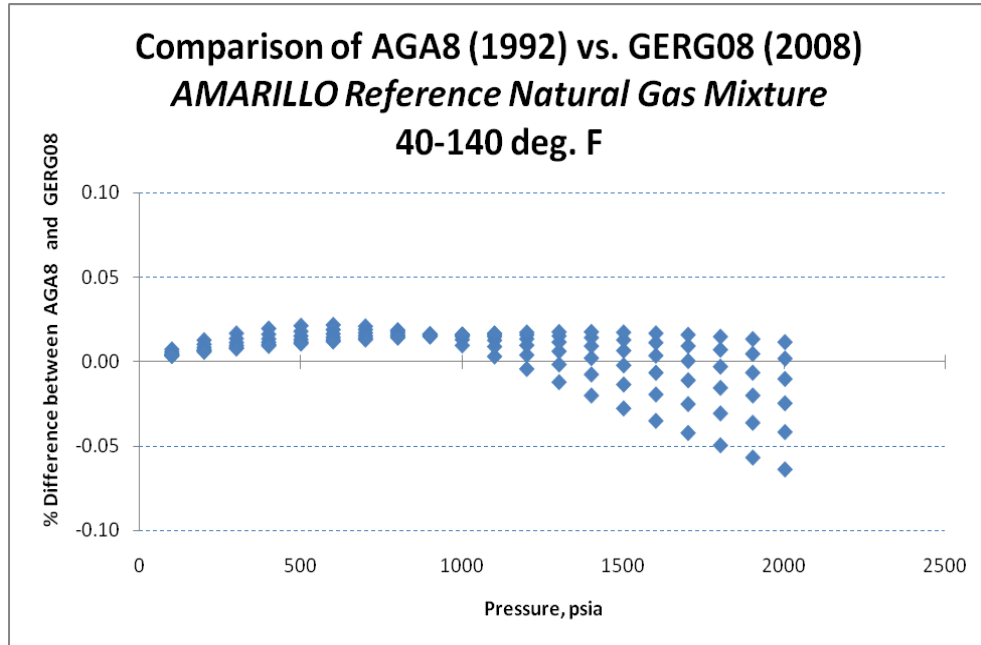


Figure 6. Amarillo Gas Uncertainty - Difference between AGA8 (1992) and GERG(2004/2008)

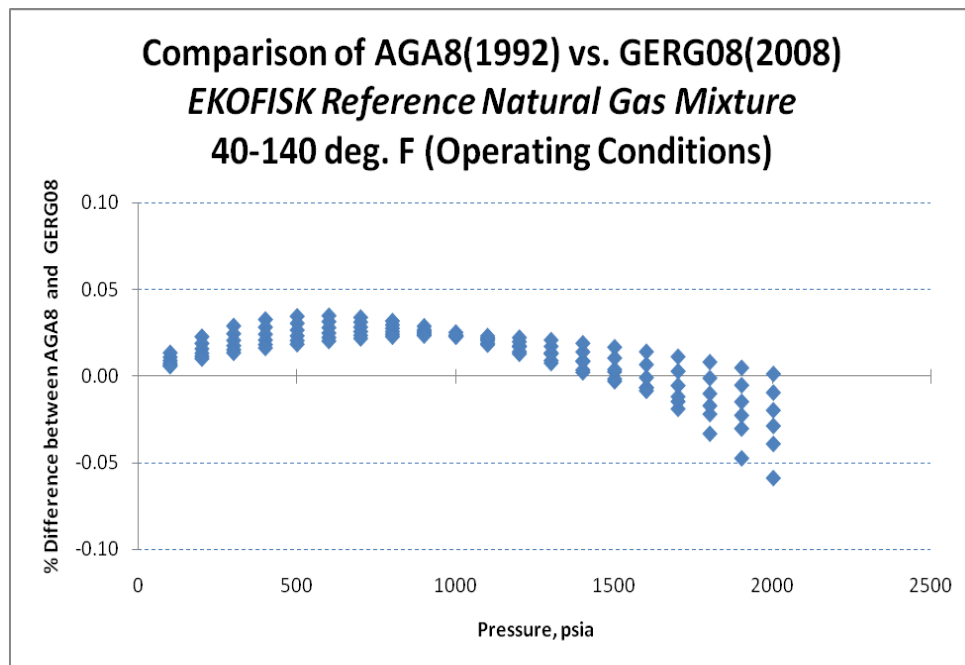


Figure 7. Amarillo Gas Uncertainty - Difference between AGA8 (1992) and GERG(2004/2008)

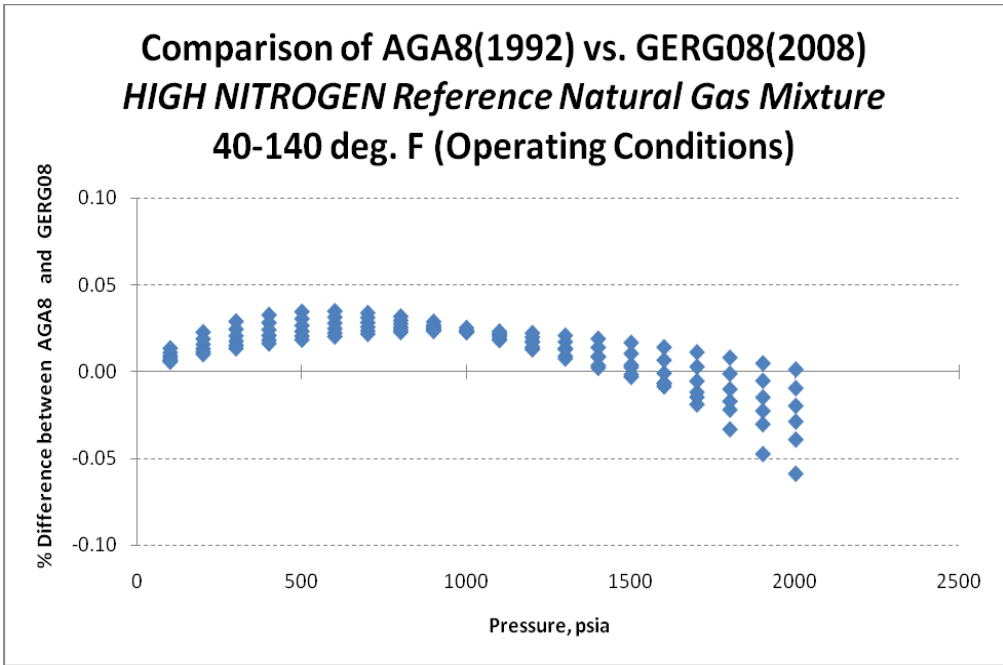


Figure 8. High Nitrogen Gas Uncertainty - Difference between AGA8 (1992) and GERG(2004/2008)

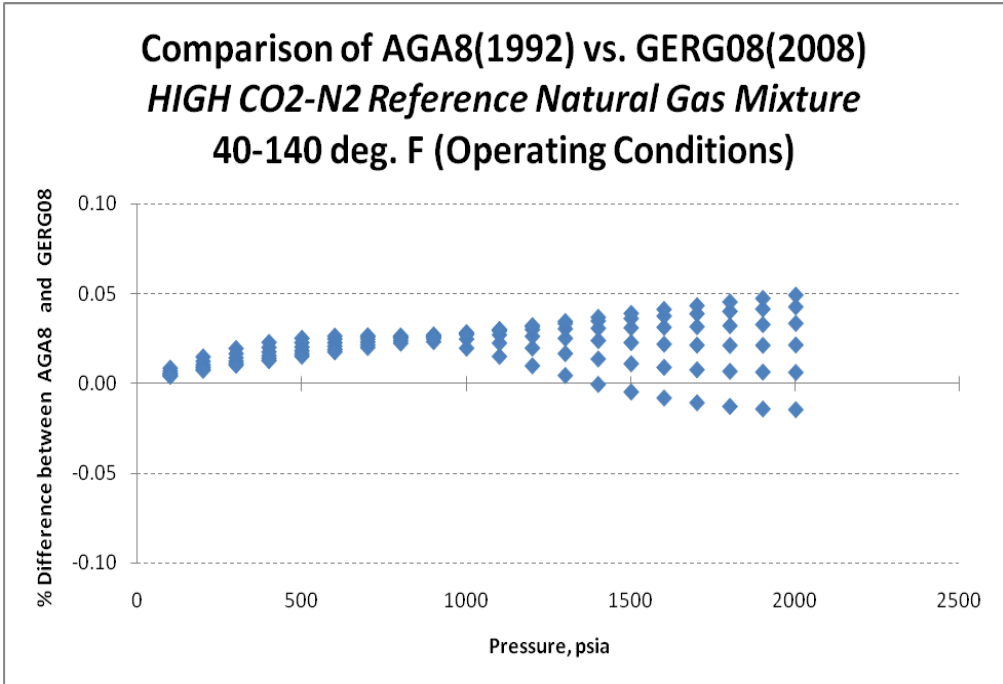


Figure 9. High CO2-N2 Gas Uncertainty - Difference between AGA8 (1992) and GERG(2004/2008)

The mixture reference data list above and listed in reference 25 are your best available data for measurement validation purposes. Arbitrarily collected samples will suffer from a wide range of problems which will create all sorts of unpleasant measurement issues. The adage, garbage in - garbage out is used for good reason. Users need to be aware of the cautions offered for guidance in the front-end of AGA Report no. 8 and the associated remedies.

SOUND SPEED

Speed of sound information is used in many gas measurement applications. Its uses include gas measurement with ultrasonic meters, transducer and vibrating element calibrations, path performance monitoring for single and multipath ultrasonic meters, and critical venture calculations.

The expression used to determine the sound speed, C, is derived from the behavior of an infinitesimal pressure wave disturbance (sound wave) in a gas from the energy and mass balance equation. Equation 6 shows that the sound speed is proportional to the square root of the change in density with respect to a change in the pressure at constant entropy, $(\partial P/\partial \rho)_s$. It is not practical to make measurements at such conditions. Equation .6 is expressed in terms of the relationship of the sound speed to the compressibility factor as shown in equation 7.

$$(.6) \quad C = \left[\frac{g_c}{M_r} \left(\frac{\partial P}{\partial \rho} \right)_s \right]^{\frac{1}{2}}$$

$$(.7) \quad C = \left[\frac{C_p}{C_v} \frac{RT}{M_r} \left(Z + \rho \left(\frac{\partial Z}{\partial \rho} \right)_T \right) \right]^{\frac{1}{2}}$$

where

g_c = gravitational constant
 M_r = molecular mass
 P = pressure, absolute
 T = absolute temperature
 ρ = density
 S = entropy
 C_p = Constant pressure heat capacity
 C_v = Constant volume heat capacity
 Z = compressibility factor
 R = universal gas constant

The expressions to compute the sound speed are involved and go beyond the intent of this paper. Basically equation .7 shows that the sound speed is a function of the temperature, ratio of constant pressure and constant volume heat capacities, gas constant, molecular weight, compressibility factor, density, and the change in the compressibility factor with respect to density at constant temperature. Computer programs should be used to make sound speed calculations. The performance should be validated against reference data.

SOUND SPEED UNCERTAINTY

Experimental data on sound speed for natural gas mixtures are rare. Because of the scarcity of experimental sound speed data it is difficult to state the uncertainty with the same level of confidence as we have with compressibility factors. The experimental data for sound speed on mixtures has the fluid characteristics identified in Table 3.

Fluid	Gulf Coast	Amarillo	Rich Gas, Dry	Very Rich
Methane	96.5	90.1	83.98	74.35
Ethane	1.8	4.5	13.5	12.0
Propane	.4	.81	.94	8.25
Butanes	.2	.15	.1	3.0
Pentanes	.07	.09	.2	.6
Hexanes	.06	.03	.0	.2
Carbon Dioxide	.6	.5	.75	1.0
Nitrogen	.26	3.1	.71	.53

Table 3. Nominal compositions for sound speed reference gas mixtures.

The uncertainty of the calculated sound speed from an equation of state depends on the specific temperature, pressure and fluid composition among other quantities. The uncertainty of the calculated sound speeds for natural gas were evaluated by comparing calculated values to a limited body of experimental measurements on natural gas mixtures²⁴. The uncertainties are estimates and are based on the temperature, pressure and composition range of the measurements.

Speed of sound calculations using the method in A.G.A. Report No.10 may be summarized for various systems as follows:

- Pure methane sound speed calculations provide an average absolute deviation ranging from 0.016% to 0.05% and maximum deviations to 0.18 %. The measured data cover the range from approximately 20 °F to 140 °F and pressures to 1700 psia.
- Gulf Coast gas mixture sound speed calculations provide an average absolute deviation of 0.06% and maximum absolute deviation of 0.14 %. The measured data cover the range from approximately 20 °F to 140 °F and pressures to 1700 psia.
- Amarillo gas mixture sound speed calculations provide an average absolute deviation of 0.04% and maximum absolute deviation of 0.1 %. The measured data cover the range approximately 20 °F to 140 °F and pressures to 1700 psia.
- Rich (Ekofisk type) gas mixture sound speed calculations provide an average absolute deviation of 0.05% and maximum deviation of 0.15 %. The measured data cover the range from approximately 20 °F to 140 °F and pressures to 1700 psia.
- Very rich gas mixture speed of sound calculations provide an average absolute deviation of 0.17% and maximum absolute deviations of 0.83 %. The measured data cover the range from approximately 20 °F to 140 °F and pressures to 1700 psia. The high deviations may suggest obtaining additional experimental data to verify whether condensation is an issue.

For a high degree of accuracy for different fluids, it may be advisable to validate the results of speed of sound calculations. For example, you may wish to obtain independent sound speed measurements or make calculations using another method or both. In principle, the fact that two methods agree does not validate the performance of either method or the data that they may be compared against. Multiple measured data sets and calculation methods help provide a basis to reconcile the magnitude of uncertainty among the data and methods.

CONCLUSIONS

Gas compressibility has a long and important history for the gas industry. Real gas behavior cannot be ignored due to its significant impact on volumetric, mass and energy flow calculations. The gas compressibility factor and sound speed are not simple quantities to calculate. The complexity of the AGA Report No.8 , AGA Report No.10 and ISO 20765 methods for the compressibility factor and speed of sound is the price of high accuracy, reduced uncertainty in gas measurement and advanced measurement methods. Fortunately, computers make the task of computing the compressibility factor and sound speed relatively trivial for the industry user. A.G.A. Report No.8, A.G.A. Report No. 10, and ISO 20765 provide comprehensive methods that offer high accuracy and utility to the gas industry. Lastly, and importantly, experimental reference data are the foundation of high accuracy correlation development, equation of state performance evaluation, and flow measurement uncertainty determination for both the laboratory and the field.

Acknowledgements

The author gratefully expresses his appreciation to Mr. R. Goodenough for the helpful discussions and materials and to Mr. Ronald Beatty (Ron) for his friendship and discussions for over thirty years. His friends at ISHM miss him and will always remember him. Lastly, I would like to thank Mr. Norman Englade of Shell, Motiva, for his many helpful discussions and insights.

References

1. Starling, K.E., and Savidge, J.L., Compressibility Factors of Natural Gas and Other Related Hydrocarbon Gases, American Gas Association, Transmission Measurement Committee Report No.8, and American Petroleum Institute, MPMS Chapter 14.2, Second Edition, 1994
2. International Standard, 12213-2 Natural Gas – Calculation of Compression Factor, Part 2: Calculation using molar-composition analysis.
3. Zimmerman, R.H., Manual for the Determination of Supercompressibility Factors for Natural Gas, PAR Project NX-19, 1963
4. American Petroleum Institute, MPMS, Chapter 14.2
5. GPA 2172-96, Calculation of Gross Heating Value, Relative Density, and Compressibility Factor for Natural Gas Mixtures from Compositional Analysis, 1996
6. Speed of Sound in Natural Gas and Other Related Hydrocarbon Gases, A.G.A. Transmission Measurement Committee Report No.10
7. International Standard, 20765-1 Natural Gas – Calculation of Thermodynamic Properties for Transmission and Distribution Applications.
8. International Standard, 20765-2 Natural Gas – Extended Ranges of Application
9. Onnes, H.K., Commun.Phys.Lab, Leiden, Holland, 71(1901)
10. Beattie, J.A. and O.C. Bridgeman, Proc., Am. Acad. Arts. And Sci., 63,229 (1928)
11. Benedict, M., et al, J. Chem. Phys.,8,334 (1940)
12. Redlich, O. and J.N.S. Kwong, Chem.Rev., 44,233 (1949)
13. Lee, B.Bi., and M.G. Kesler, AIChE J., 21,510,(1975)
14. Peng, D.Y., and D.B. Robinson, Ind.Eng.Chem.Fund., 15,59,1976
15. Soave, G., Chem.Eng.Sci., 27, 1197 (1972)
16. Starling, K.E. and M.S. Han, Hydrocarbon Processing, 51(5), 129, (1972)
17. Pitzer, K.S., J. Am. Chem.Soc., 77, 3427 (1955)
18. Jamison, W.H. and E.L. Upp, Internal Report, Tennessee Gas Transmission Co., 1954
19. E.S.Burnett, J. of Applied Mechanics, Vol.3, No.4, 136,1936
20. Savidge, J.L. SuperZ : AGA8 Computer Program for Supercompressibility, Compressibility, Sound Speed, and Other Thermophysical Properties of Natural Gas- Software and Documentation, Internal Report, 1985
21. Savidge, J.L., Starling, K.E., and R.L McFall, Sound Speed of Natural Gas, Society of Petroleum Engineers Gas Technology Symposium, Dallas, TX, 1988
22. Savidge, J.L. and J.J.S. Chen, Evaluation of Critical Flow Factors and Speed of Sound Methods for Gas Measurement, Proc. 4th IGRC, Tokyo, 1989
23. J. L. Savidge, Report to AGA Transmission Measurement Committee Task Group 13 on A.G.A. Report No. 10, Speed of Sound, 2000 AGA Operations Conference, May 7-9th, Marriott, Denver, 2000
24. J. L.Savidge, Report to AGA Transmission Measurement Committee Task Group 13 on A.G.A. Report No. 10 , Speed of Sound, 2000 AGA Operations Conference, May 7-9th, Marriott, Denver, 2001
25. Savidge, J.L. et al Technical Reference Document for AGA Report No.8, (GRI –93/0181, March 1995)